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Understand Human Walking through a 2D Inverted Pendulum Model

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Abstract— **This paper gives some macroscopic understandings on human walking about the limitations on walking speed and step length, the reachable region, capture region, and disturbance recovery through a 2D inverted pendulum model. Our concern is the most basic problems in human walking, such as what are the limitations on walking speed and step length, how people change speed during step-to-step transition, and how people prevent a fall. The concept of walking orbit is proposed as a tool to study these problems. It describes the walking motion in the state space under walking constraints, giving us an intuitive way to study human walking during a step and switch between steps. The model has a point mass on the hip and two massless legs. The two dominant control inputs, hip and ankle actuation are idealized into a free determined foot placement and an impulsive push off. Based on this model, some quantitative and qualitative analysis are given, leading to some macroscopic understandings on human walking. Although this paper does not talk about any details on how to realize the control for a real biped robot, it may serve as a helpful guide for biped robot design and control in the future.**

I. Introduction

Walking upright is considered as one of the most important characteristics of human beings which differs from other animals. Walking seems so easy for us today, but as a result of evolution, it takes millions of years for ancient human to master this skill. It reminds us walking is not so simple as it looks like. Even today with so many advanced technologies, walking is still a challenging task for biped robots. To date, no robots can walk as sable and as efficient as us. So many researchers are continually exploring the secrets behind human walking and trying to make more efficient biped robots.

One of the attempts is based on passive walking which was pioneered by McGeer [1]. Passive walking refers to a class of bipedal machines that can walk down a gentle slope by gravity with no external control. Several fancy passive walking machines have been created [2-3], including a straight-legged version, a 2D version with knees, and especially a 3D version with knees and arms [3] that can walk downhill in a very humanlike way. To understand the mechanism of passive walking, some mathematical models were built [4]. An extreme case was the simplest walker [5], which has a point mass hip, two massless legs, along with two point-feet with infinitesimal mass. Through the simplest walker model, the relationship between the walking gait and the slope angle is found. This encourages the following researchers to investigate human walking through simple models.

A disadvantage of passive walker is that it cannot walk on level ground and the basin of attraction is quite narrow. So people add actuation, mostly hip and ankle actuation, to replace gravity to enable it walk on level ground. Typical examples are the Cornell biped and the Delft biped [6], which use primitive control once per step and have motions very close to a ramp-walking machine. An upgraded version, the Cornell ranger has created a walking record of 65 km with the cost of transport 0.28, which is comparable to a real human [7]. At the same time, researchers also try to modify the simplest walker model by adding controls, such as leg swing control, push off, and flywheel to study various aspects of actuated walking [8-15], including energetics, stability, basin of attraction, disturbance recovery, and so on.

However, most of the paper focus on the control details while some basic problems are not answered. We intend to study human walking through simple models. Our concern is the most basic problems in human walking, such as what are the limitations on walking speed and step length, how people change speed during step-to-step transition, and how people prevent a fall. The 2D inverted pendulum model in [15] will be used. Although many more complicated models have been studied including the one with an upper body [16], toed feet [17], the five-link model [18], and the seven-link model [19], they are of less concern to us since we are most interested in the motion of the center of mass (COM).

In this paper, four aspects are investigated for the simple walking model, including the limitations on walking speed and step length, the reachable region, the capture region, and the disturbance recovery. The concept of walking orbit is proposed, which describes the walking motion in the state space under walking constraints. It gives us an intuitive way to study human walking during a step and switch between steps. The concept of capture region is not new [10], which initially refers to the location where the robot needs to place its foot to come to a complete stop, here we will use it to represent the region in the state space from which it is possible to stop. Besides, we use the term reachable region to represent the region in the state space where it is possible to reach from a given state, which is very similar to the reachable workspace concept for a manipulator [20] and is also an inverse concept to the controllable region in [15]. An interesting observation is

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that the reachable region and controllable region are exactly the same for a given orbit (credit to Prof. Andy Ruina). For disturbance recovery, we will study a simple case of stop from an initial push as in [10]. With the help of walking orbit, some quantitative and qualitative analysis are given. The main conclusions are summarized as follows:

(1) Limitations on walking speed and step length: Higher speed can be achieved with smaller steps. The walking speed is limited by 3.16 m/s and the step length should be smaller than 1.5 m which is limited by the constraint of walking that there should always be at least one foot in contact with the ground.

(2) Reachable region: By taking bigger steps, people can change the walking speed in a bigger range. The two-step reachable region covers more than 90% of the walking zone, which supports the conclusion in [15] that two steps is enough.

(3) Capture region: People can stop in one step when the mid-stance velocity is smaller than 2.5 m/s, and stop in two steps when the mid-stance velocity is smaller than 2.98 m/s.

(4) Disturbance recovery: When the push speed is bigger than 2.5 m/s, it will need two or more steps to stop and the step length should keep small to prevent a fall. If you want to stop from a push, you will need to swing your leg two times as fast as the push speed.

The numbers above may be not accurate for a real human due to the ideal model we used, but it gives some macroscopic understandings on human walking and may serve as a helpful guide for biped robot design and control in the future.

The remainder of this paper is organized as follows. Section II describes the walking model. Section III introduces the concept of walking orbit. The limitations on walking speed and step length are studied in Section IV, reachable region in Section V, capture region in Section VI, and disturbance recovery in Section VII. Conclusion is given in Section VIII.

II. Walking Model

The model used in this paper is a minimal biped model in [15], which has a point mass on the hip and two massless legs. It has no knees or feet and the legs are incompressible. Only walking will be studied in this paper. The walking model is a hybrid system which consists of a continuous phase during leg swing and a switch phase during heel strike. The two dominant control inputs, hip and ankle actuation are idealized into a free determined foot placement and an impulsive push off.

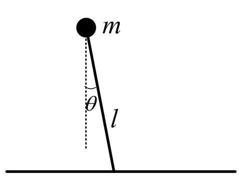

Figure 1. Walking model during continuous phase.

During the continuous phase, the stance leg acts as an inverted pendulum as shown in Figure 1, the swing leg motion does not influence the stance leg dynamics since the swing leg is massless. The motion is governed by the equation as follows

$$\ddot{\theta} = g\sin\theta / l \tag{1}$$

where θ is the stance leg angle, and l is the length of the leg.

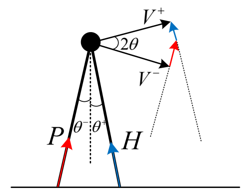

Figure 2. Walking model during switch phase.

During the switch phase, there is an instantaneous instant of double stance as shown in Figure 2. Two impulsive forces are imposed from the ground along the two legs, where one is the heel strike H which acts on the leading leg and the other is the push off P applied to the trailing leg. The impulses result in a sudden change on the velocity of the COM. The switch equation is determined by principle of linear momentum, that is

$$m\vec{V}^{+} - m\vec{V}^{-} = \vec{P} + \vec{H} \tag{2}$$

From which it can be obtained that

$$\dot{\theta}^{+} = \dot{\theta}^{-}\cos 2\theta^{-} + P\sin 2\theta^{-} / ml \tag{3}$$

After switch, the former stance leg becomes the new stance leg, so we have $\theta^{+} = -\theta^{-}$.

III. The Concept of Walking Orbit

In this section, the concept of walking orbit will be introduced, which is the foundation of this paper.

First, some walking constraints will be discussed. Since we only focus on walking in this paper which means no flight phase as in running, the stance leg can never leave the ground, which puts some constraints on the system. First, during the continuous phase, the leg compression should always be nonnegative to ensure that the walker will not fly in the air. Second, during switch phase, the push off and heel strike impulses should be nonnegative because of the ground force direction. These two constraints can be expressed by a set of inequations.

Constraint 1 (continuous phase): leg compression nonnegative

$$mg\cos\theta - ml\dot{\theta}^2 \geq 0 \tag{4}$$

It can be simplified as

$$\dot{\theta}^2 \leq g\cos\theta / l \tag{5}$$

Constraint 2 (switch phase): push off /heel strike nonnegative

Consider two extreme cases when the collision happens with only push off or heel strike as shown in Figure 3.

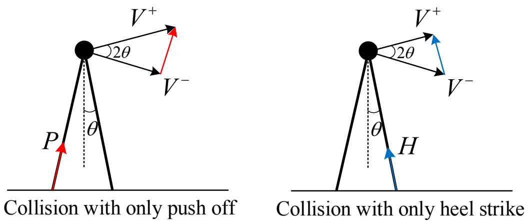

Figure 3. Collision with only push off /heel strike .

With only push off, it gives the maximum velocity after collision

$$\dot{\theta}^+ = \dot{\theta}^- / \cos 2\theta \tag{6}$$

With only heel strike, it gives the minimum velocity after collision

$$\dot{\theta}^+ = \dot{\theta}^- \cos 2\theta \tag{7}$$

Therefore, (6) and (7) set the upper and lower bound on the velocity after switch, respectively. So Constraint 2 can be expressed as follows

$$\dot{\theta}^- \cos 2\theta \le \dot{\theta}^+ \le \dot{\theta}^- / \cos 2\theta \tag{8}$$

These two constraints actually reflect the limitations on the walking motion during one step (continuous phase) and between steps (switch phase). Constraint 1 sets an upper bound on the walking speed, indicating that the walker cannot walk too fast otherwise it will leave the ground, while Constraint 2 sets boundaries on the speed change between steps. The speed change cannot be arbitrary since the ground reaction force has a direction, namely it can only push the walker but not pull.

We can express the constraints in a more vivid way by using pictures. The model has only two state variables, that is, the stance angle θ and angular rate $\dot{\theta}$, which can be expressed in a 2D coordinate plane. We will use θ as the horizontal axis and $\dot{\theta}$ as the vertical axis. Unlike the commonly used nondimensional skill, the parameters here will be selected close to a real human as $l = 1m, g = 10m/s^2$ so that the data can serve as an approximate reference for human walking.

With Constraint 1, the walking space is constrained in an approximate ellipse with the boundary $\dot{\theta} = \pm\sqrt{g\cos\theta / l}$ as shown in Figure 4. The status of the walker at any moment can be represented by one point in the ellipse. During one step, the motion of the walker will follow (1). However, it does not give an explicit relationship between the two system variables. We can obtain their relationship from the conservation of mechanical energy, that is, $E = ml^2\dot{\theta}^2/2 + mgl\cos\theta$ is conserved during one step, from which we can get $\dot{\theta} = \pm\sqrt{(2E - 2mgl\cos\theta)/(ml^2)}$. This defines the walking trajectory in the walking plane during one step, which we call a walking orbit. Each orbit has the same mechanical energy. Some walking orbits are given in Figure 4. Each orbit has a direction which represents the states moving direction with time.

Among the walking orbits, there is a special orbit which passes through the origin. It has zero mid-stance velocity and is defined by $\dot{\theta} = \pm\sqrt{2g(1-\cos\theta)/l}$, which divides the walking space into two parts. The orbit in the green part is the high-energy orbit which can pass through the upright point, and the orbit in the yellow part is the low-energy orbit which cannot pass the upright point. We will call the green area as the walking zone and the yellow area as the stopping zone. During steady walking, the walker moves on the orbit in the walking zone, where the top and bottom half represents two walking directions. To stop, the walker should lower down the energy to the stopping zone and then put the trailing leg down when the body speed achieves zero.

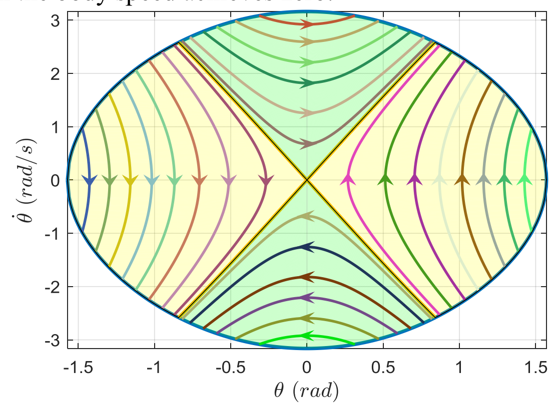

Figure 4. Walking space and walking orbit.

Now consider the transition between steps. During transition, the walker can keep in the same orbit or switch to another orbit. However, the switch is limited by Constraint 2. From the initial orbit, there is a boundary on the orbits that it can switch to. When the heel strike is zero, it can switch to the highest-energy orbit $\dot{\theta}^+ = \dot{\theta}^- / \cos 2\theta$. When the push off is zero, it can switch to the lowest energy orbit $\dot{\theta}^+ = \dot{\theta}^- \cos 2\theta$. This gives two boundary lines $\dot{\theta} / \cos 2\theta$ and $\dot{\theta} \cos 2\theta$ along the initial orbit as shown in Figure 5.

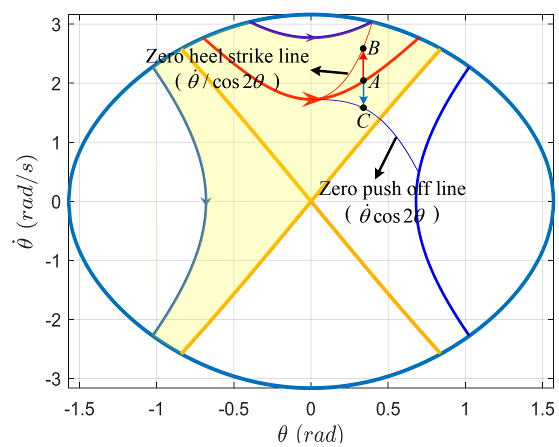

Figure 5. Step-to-step transition boundaries of walking.

As shown in Figure 5, the red line is the initial orbit. Switch happens in the right part of the orbit. After switch, the stance angle will reverse, and the angular rate should change within the two boundary lines. For example, if the transition happens in point A, then it can only switch to the orbit that passes between B and C. The next step will start on the left part of the same or another obit. If transition happens at different stance angle, the orbits it can switch to are different. Generally, with longer step, it has more switch options which means the switch space is bigger. But when the step length is bigger than a certain value, the higher-energy orbit it can switch to will decrease. The yellow area shows the maximum switch area of the red orbit.

IV. LIMITATIONS ON WALKING SPEED AND STEP LENGTH

In this section, the relationship between walking speed and step length will be studied and some limitations will be addressed. We will focus on steady walking, so only the walking zone will be considered.

As shown in Figure 6, for a given stance angle, there is a feasible orbit range which is determined by the upper and lower bound of the walking zone (above that is the running zone and below is the stopping zone). This gives the speed range when walking with a given step length.

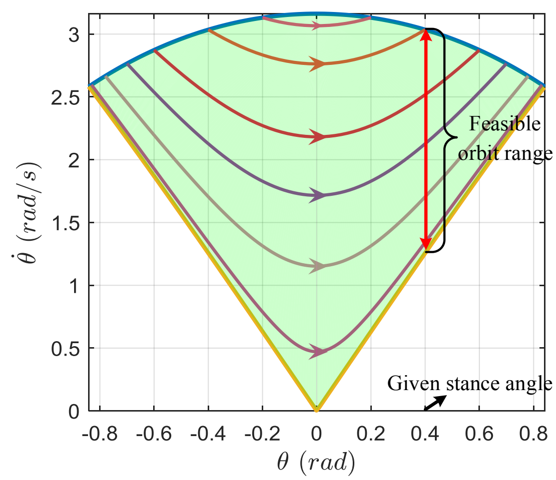

Figure 6. Feasible orbit range for a given stance angle.

For a given step length $2l\sin\theta$, the walking speed is $V = 2l\sin\theta / T$, where T is the time duration of the step. Figure 7 shows the walking speed range for different step lengths.

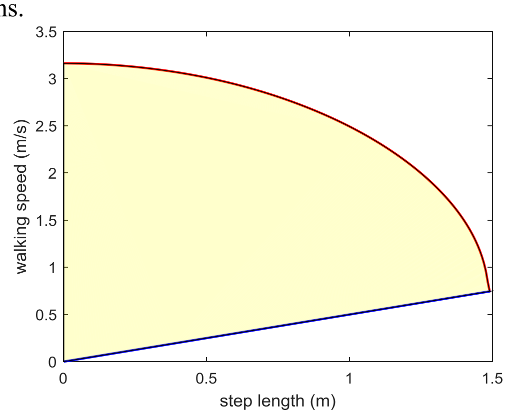

Figure 7. Walking speed range for different step lengths.

From Figure 7, it can be observed that the walking speed has a broader range for smaller steps and higher speed can be achieved with smaller step length. This is consistent with our experience that we will take shorter steps when walk faster. We also find two interesting numbers. One is the highest walking speed 3.16 m/s, and the other is the maximum step length 1.5 m. These two numbers reflex some inherent limitations on human walking speed and step length. The walker cannot walk faster than 3.16 m/s and the step length should be smaller than 1.5m, otherwise it will transit to running. The two numbers give an approximate estimation on the maximum human walking speed and step length.

V. REACHABLE REGION

In this section, the reachable region will be studied. It tells the region in the state space where it is possible to reach from a given state.

As discussed in Section III, there is a boundary on the reachable orbits during step transition. The upper bound is the zero heel strike orbit and the lower bound is the zero push off orbit. This defines the one-step reachable region. By starting from the upper and lower bound orbit and repeat the process, we can also obtain the two-step reachable region.

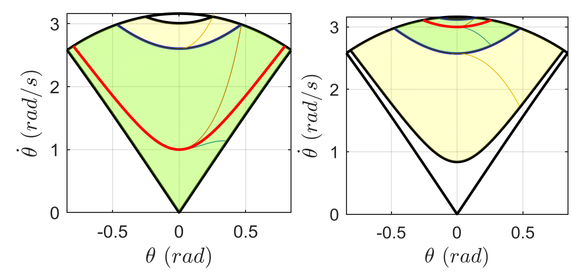

Figure 8. Examples of reachable region.

Figure 8 shows two examples of the reachable region. The red orbit is the starting orbit. The green area is the one-step reachable region and the yellow area plus the green area is the two-step reachable region. Starting from different orbits, it will have different reachable region. The reachable region is an inverse concept to the controllable region in [15] and it can be verified that the reachable region and controllable region are exactly the same for a given orbit.

We are interested in the percentage of the reachable region for different starting orbits. There are two ways to calculate the percentage. One is to use area, which calculates the percentage of the reachable region to the entire walking zone. The other is to use length, which calculates the percentage from the length of the reachable mid-stance velocity to the length of the entire feasible mid-stance velocity. The results are shown in Figure 9, where the horizontal axis shows the mid-stance velocity of the starting orbit. The results are very similar for the two computing methods. It can be seen that the one-step reachable region covers more than 80% of the walking zone but has a sharp drop when the mid-stance speed is more than 2.5 m/s. The two-step reachable region covers over 90% of the walking zone and drops at the mid-stance speed of 2.98 m/s which is very close to the top reachable speed. This supports the conclusion that two steps is enough [15], that is, if you can reach a target velocity at all, you can probably do so in two steps.

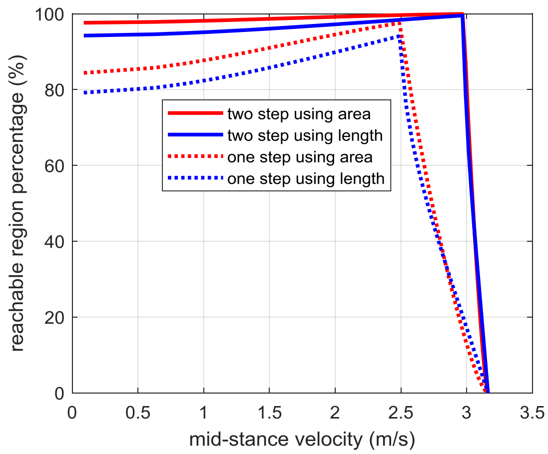

Figure 9. One -step and two-step reachable region percentage.

Another way to describe the reachable region is to use mechanical energy. From an initial energy, the energy it can switch to in the next step also has a boundary. Figure 10 shows the energy switch range with $m = 1kg$. The horizontal axis shows the energy of the former step and the vertical axis is the energy of the next step. The black line has the same energy before and after switch. The red line gives the upper bound of the energy after switch while the blue line represents the lower bound.

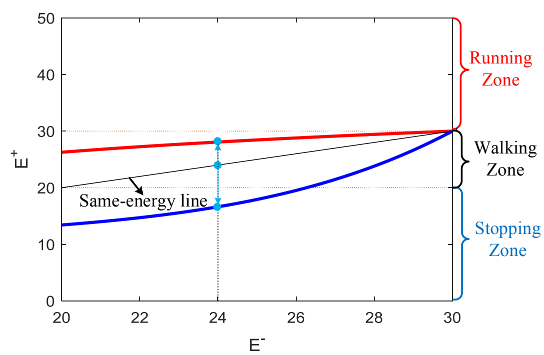

Figure 10. Energy switch range during step-to-step transition.

VI. Capture Region

This section will study the capture region, which describes the region in the state space from where it is possible to stop. It is a common sense that we can stop easily when walk slowly, but if we walk faster, we will need more steps to stop. Here we will give quantitative explanations for this phenomenon.

Still using the walking orbit, the transition from walking to stop is a process of energy decay, which is a transition from the walking zone to the stopping zone. Therefore, for a walking orbit, if its zero push off line can reach the stopping zone, then it can stop in one step. If the zero push off line of the one-step lowest reachable orbit can reach the stopping zone, then it can stop in two steps.

We use n-step capture region to describe the region from where it can stop in n steps. As shown in Figure 11, the blue orbit shows the upper bound of the one-step capture region. It has the highest mid-stance velocity of 2.5 m/s, which is 79% of the top reachable speed. The red orbit is the upper bound of the two-step capture region with the highest speed 2.98 m/s, which is 94% of the top walking speed. It also supports the conclusion that two steps is enough [15], that is, if you can stop at all, you can probably do so in two steps.

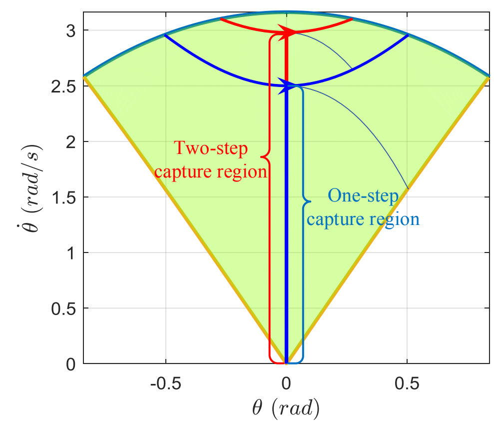

Figure 11. One-step and two-step capture region.

VII. Disturbance Recovery

To study the ability of recovery from a disturbance, we will study a simple case: stop from an initial push. As shown in Figure 12, the walker is initially standing still, then with a sudden push, the COM gets a velocity and then it tries to stop in one or multiple steps.

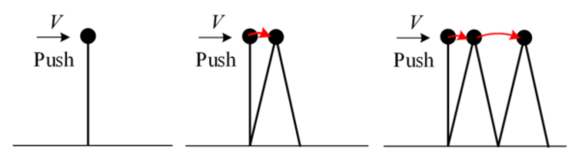

Figure 12. Push to stop.

Informally, we use the word push speed to represent the speed of the COM after push. Figure 13 shows the one-step and two-step stop line. It indicates the step length needed to take to stop in one step or two steps. When the push speed is smaller than 2.5 m/s, it is able to stop in one step. To stop, the step length should be big enough. It can be observed that for harder push, it will need bigger step length to stop. However, when the push speed is bigger than 2.5, it will need two or more steps to stop and the step length should keep small to prevent a fall (or you may jump).

Another important factor to stop is the leg swing. By taking steps to stop, it needs the cooperation of the swing leg to reach the foot placement before falling. To stop, the swing speed should be fast enough. Figure 14 shows the average foot swing speed $2\theta / T$ when taking steps at the stop line, where θ is the stance angle on the stop line and T is the time duration of the step. On one hand, it indicates that the robot should have the swing ability of 6 rad/s to be robust enough to disturbances. On the other hand, it can be seen the resulted line is approximately with a gradient of 2. So generally speaking, if you want to stop from a push, you will need to swing your leg two times as fast as the push speed.

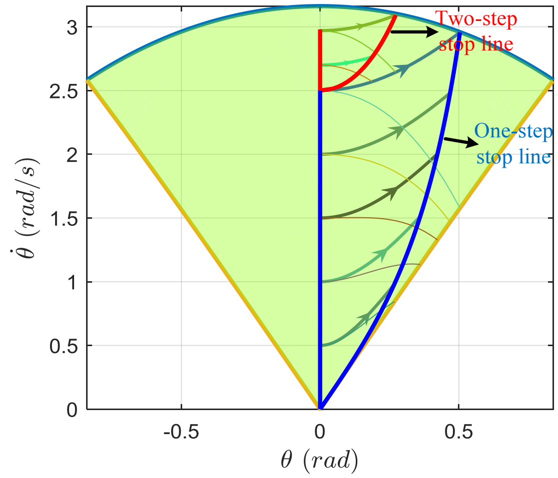

Figure 13. One-step and two-step stop line.

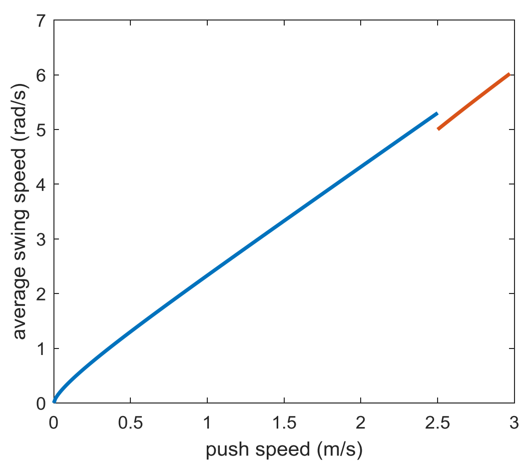

Figure 14. Swing speed needed to stop for a given push speed.

VIII. Conclusion

This paper gives some macroscopic understandings on human walking through quantitative and qualitative analysis on a simple walking model, which may be meaningful for biped robot design and control. For example, longer step helps change speed quickly, but the robot should keep the step length not too big to prevent a fall. The swing speed is very important to a biped robot. Faster swing means higher walking speed and stronger robustness to disturbances. And to stop from a push, the robot needs to swing its leg two times as fast as the push speed. In the future, we will extend the results to a 3D inverted pendulum model.

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